Function Theory of a Complex Variable - E2

Dr David Croydon
croydon@acs.i.kyoto-u.ac.jp
www-an.acs.i.kyoto-u.ac.jp/∼croydon

First Semester 2018/19
Thursdays 2nd period (10.30-12.00)
Room 3D, Academic Centre Building, North Wing
Yoshida South Campus

Course overview

Building upon a basic knowledge of calculus, this is an introductory course to the function theory of one complex variable, i.e. an introduction to complex analysis. The goal is to understand the fundamentals of holomorphic and meromorphic functions, which are dealt with through Cauchy's integral formula. The purpose of this course is not only to understand rigourous theories, but to obtain some skills about the residue calculus. The theory for complex functions is not only beautiful in a mathematical sense, but also very useful in applied fields, e.g. physics, engineering and medical sciences, etc. Almost all the mathematical theories in this course are rigorously dealt with, and some examples related to physics are also explained. An additional goal of this course is to give a chance to students to present and discuss mathematics in English.

Format

Principally this will be a lecture course. There will be three exercise sheets, with the first handed out in the second class. Students should submit their answers the following week for grading. Students will also be asked to present/discuss solutions to some exercises in class.

Evalation

The overall mark for the course will incorporate the following:

20% for homework;

10% for presentation in class;

70% for the final exam.

Content

The course will cover the following topics, each taking approximately two or three weeks.

1. Complex numbers, the complex plane and the Riemann sphere.

2. Differentiation of complex functions, holomorphic functions and the Cauchy-Riemann equation, etc.

3. Power series and analytic functions.

4. Integration of complex functions, the Stieltjes integral and Cauchy's integral formula.

5. Fundamental theories for holomorphic functions.

6. Singularities and residues, the Laurent expansion and the residue calculus.

References

All the relevant content will be covered in the lectures, but additional background reading and exercises can be found in the following books.

L. V. Ahlfors, *Complex analysis*, McGraw-Hill, 1979

J. B. Conway, *Functions of one complex variable*, Springer-Verlag, 1978

E. M. Stein and R. Shakarchi, *Complex analysis*, Princeton, 2003